

## Tutorial 9.

### Preliminary:

Interest rate of return:

Net cashflows  $C_0, C_1, \dots, C_n$  at time  $t_0, t_1, \dots, t_n$ .

$$\sum_{k=0}^n C_k \cdot v^{t_k} = 0. \quad \text{because} \quad C_k = \underset{\substack{\uparrow \\ \text{cash in}}}{A_k} - \underset{\substack{\uparrow \\ \text{cash out}}}{B_k}.$$

the present values are equal

$$A_0 + A_1 v^{t_1} + \dots + A_n v^{t_n} = B_0 + B_1 v^{t_1} + \dots + B_n v^{t_n} \Rightarrow \sum_{k=0}^n C_k \cdot v^{t_k} = 0.$$

$$\sum_{k=0}^n C_k v^{t_k} = C_0 + C_1 (Hi)^{-t_1} + C_2 (Hi)^{-t_2} + \dots + C_n (Hi)^{-t_n} = 0$$

$$\Rightarrow (Hi)^{t_n} [C_0 + C_1 (Hi)^{-t_1} + \dots + C_n (Hi)^{-t_n}] = 0$$

$$C_0 (Hi)^{t_n} + C_1 (Hi)^{t_n - t_1} + \dots + C_n (Hi)^0 = 0$$

$$\Rightarrow \sum_{k=0}^n C_k (Hi)^{t_n - t_k} = 0$$

5.1.1.

(a)  $t_1 = 1, t_2 = 2, A_0 = 0, A_1 = 2.3, A_2 = 0, B_0 = 1, B_1 = 0, B_2 = 1.33$ .

Set up the equation at  $t_2$ .

$$C_0 = A_0 - B_0 = -1, \quad C_1 = A_1 - B_1 = 2.3, \quad C_2 = A_2 - B_2 = -1.33.$$

$$C_0 (Hi)^{t_2 - t_0} + C_1 (Hi)^{t_2 - t_1} + C_2 (Hi)^{t_2 - t_2} = 0$$

$$-(Hi)^2 + 2.3(Hi)^1 - 1.33 = 0.$$

Since  $(2.3)^2 - 4(-1)(-1.33) = -0.03 < 0$ , there is no solution for  $i$ .

(b)  $t_1 = 1, t_2 = 2, A_0 = 0, A_1 = 2.3, A_2 = 0, B_0 = 1, B_1 = 0, B_2 = 1.32$ .

Set up the equation at time 0.

$$C_0 = -1, C_1 = 2.3, C_2 = -1.32. \quad \text{then}$$

$$-1 + 2.3v - 1.32v^2 = 0. \Rightarrow v = 0.91 \text{ or } 0.833. \quad \text{then}$$

$$i = 0.1 \text{ or } 0.2.$$

5.14.

Transaction A:

$$C_0^A v_A^0 + C_1^A v_A^1 + C_2^A v_A^2 + C_3^A v_A^3 = -5 + 3.72v_A + 0 + 4v_A^3 = 0$$

$$\Rightarrow v_A = 0.79789, \Rightarrow i_A = 0.25330$$

Transaction B:

$$C_0^B v_B^0 + C_1^B v_B^1 + C_2^B v_B^2 + C_3^B v_B^3 = -5 + 3v_B + 1.7v_B^2 + 3v_B^3 = 0$$

$$\Rightarrow v_B = 0.79791, \Rightarrow i_B = 0.25328$$

A is preferable when present value  $P(C^A) > P(C^B)$ , where  $P(C^A) = \sum_{k=0}^n C_k^A v^{tk} = 0$ .

$$\text{then } P(C^A) - P(C^B) = 0.72v - 1.7v^2 + v^3 > 0 \Rightarrow 0.72 - 1.7v + v^2 > 0$$

$$\Rightarrow i < 0.1111, \text{ or } i > 0.25.$$

And B is preferable when  $0.1111 \leq i \leq 0.25$ .

5.13.

$$A_0 = 0, B_0 = 5 \times 1000 = 5000, \Rightarrow C_0 = -5000,$$

$$A_1 = 0.2 \times 1000 = 200, B_1 = 200 \Rightarrow C_1 = 0. \quad \text{now we have } \frac{200}{4} + 1000 = 1050 \text{ shares}$$

$$A_2 = 0, B_2 = 500 \times 4.5 = 2250 \Rightarrow C_2 = -2250, \quad \text{now we have } 1050 + 500 = 1550 \text{ shares}$$

$$A_3 = 1550 \times (1 + 0.25) = 8137.5, B_3 = 0 \Rightarrow C_3 = 8137.5$$

$$\text{then } C_0 v^0 + C_1 v^1 + C_2 v^2 + C_3 v^3 = -5000 - 2250v^2 + 8137.5v^3 = 0$$

$$\Rightarrow i = 0.049, \quad i''' = 0.098.$$